Lecture 7
Convolutional Neural Networks

CMSC 35246: Deep Learning

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&
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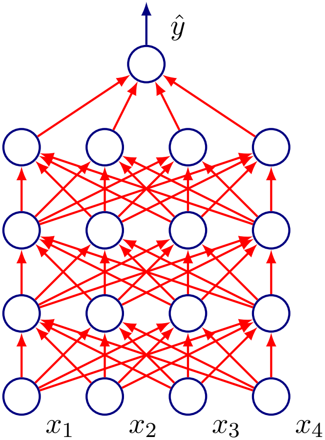
We saw before:



- A series of matrix multiplications:

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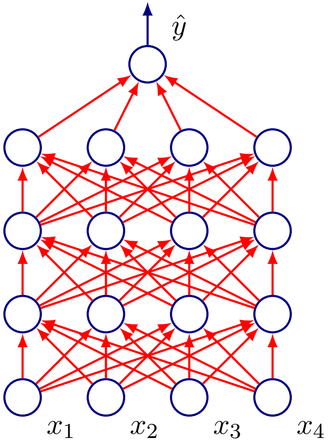

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- $\mathbf{x} \mapsto$

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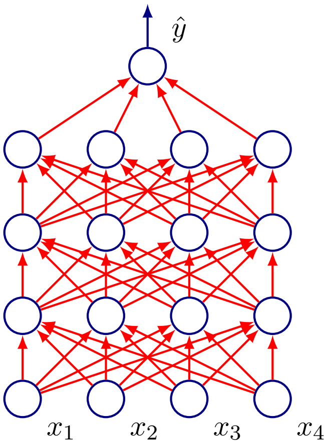

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- $\mathbf{x} \mapsto W_1^T \mathbf{x} \mapsto \mathbf{h}_1 = f(W_1^T \mathbf{x}) \mapsto$

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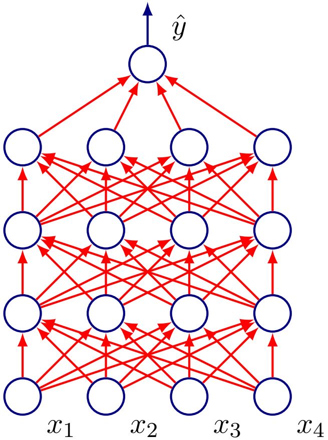

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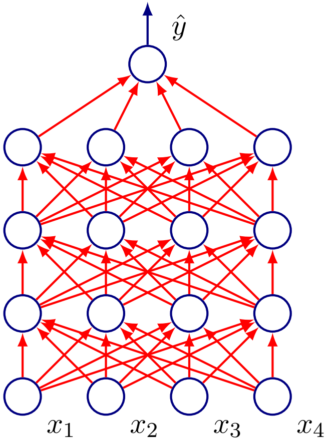

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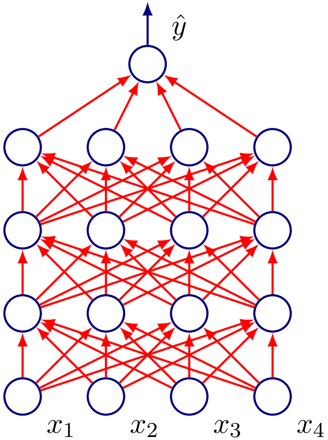

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Convolutional Networks

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  - What is convolution?

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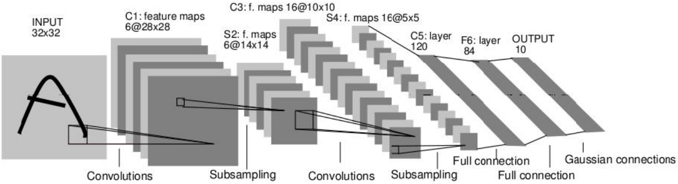
Convolutional Networks

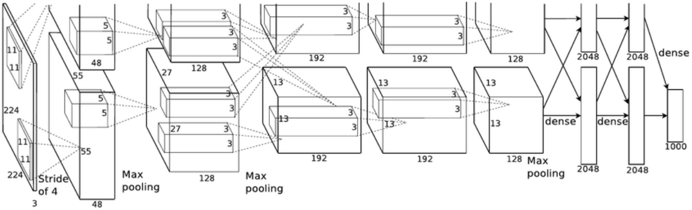
- Neural Networks that use convolution in place of general matrix multiplication in atleast one layer
- Next:
  - What is convolution?
  - What is pooling?
  - What is the motivation for such architectures (remember LeNet?)

LeNet-5 (LeCun, 1998)



- The original Convolutional Neural Network model goes back to 1989 (LeCun)

AlexNet (Krizhevsky, Sutskever, Hinton 2012)



- ImageNet 2012 15.4% error rate

Convolutional Neural Networks

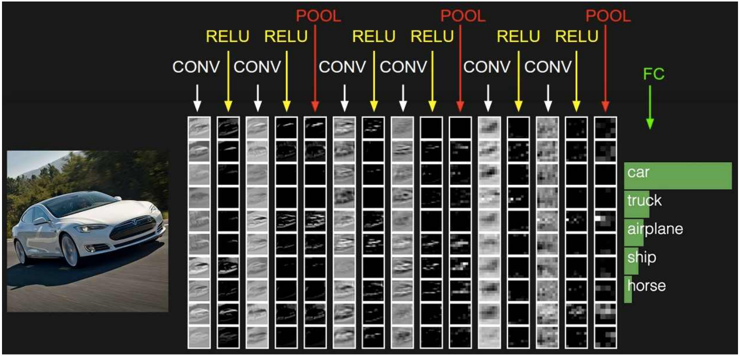
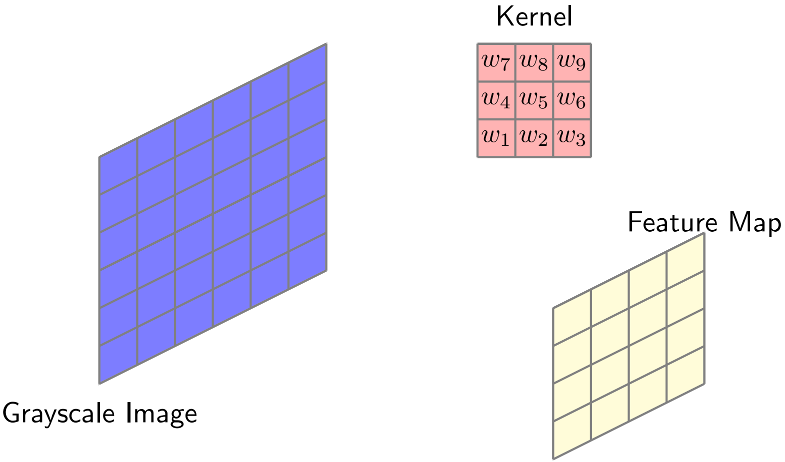


Figure: Andrej Karpathy

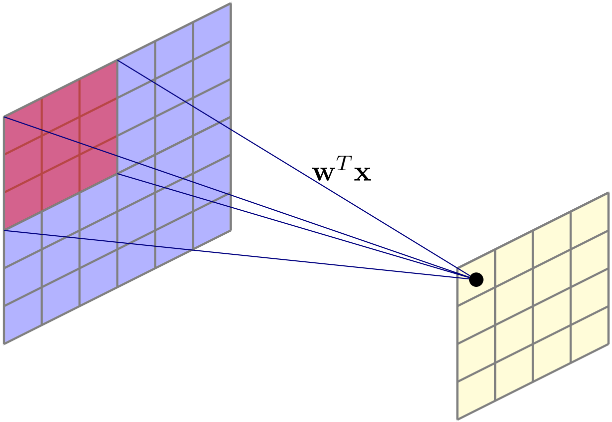
Now let's deconstruct them...

Convolution



- Convolve image with kernel having weights $\mathbf{w}$ (learned by backpropagation)

Convolution



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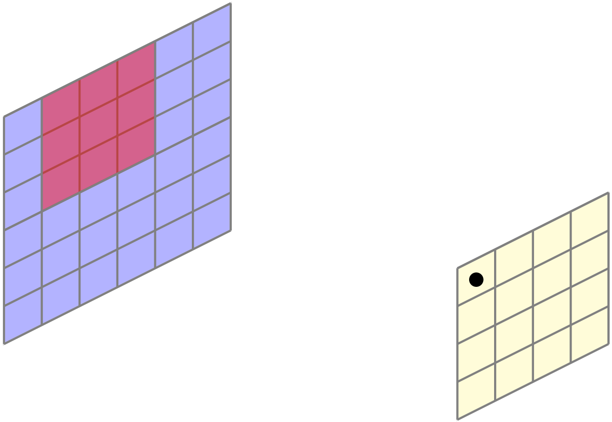

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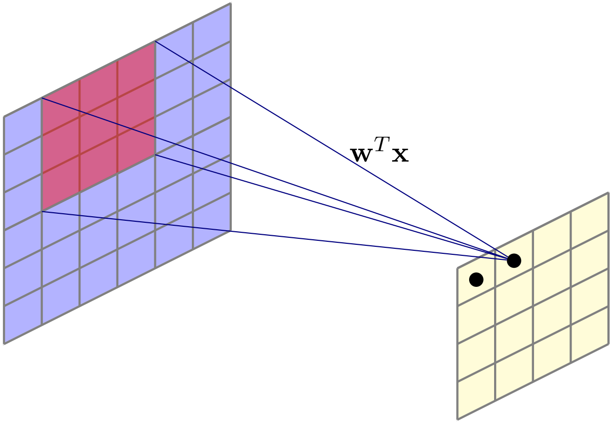

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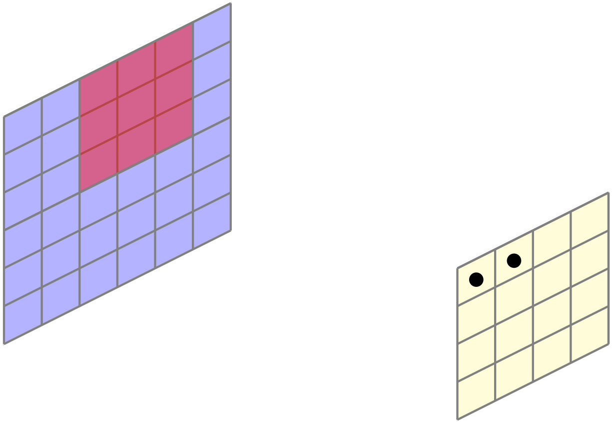

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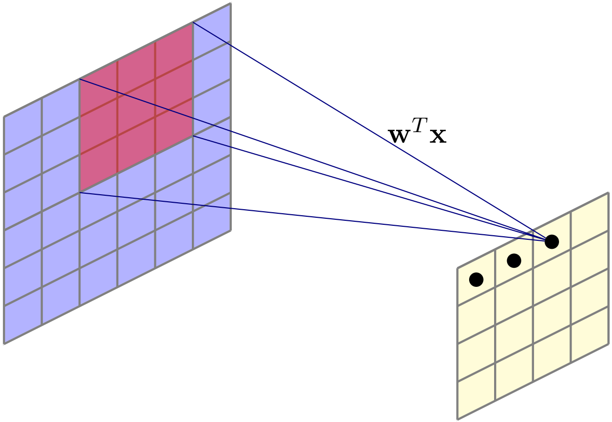

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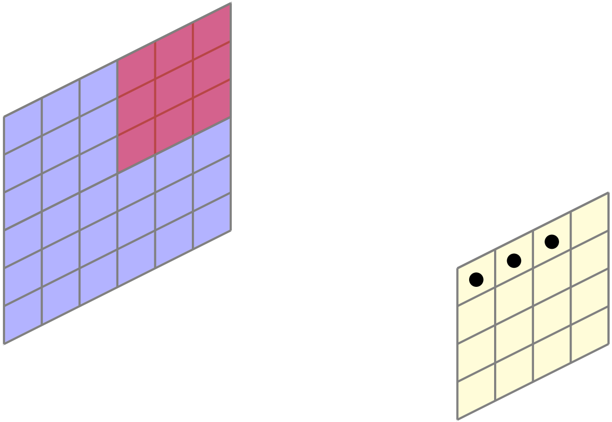

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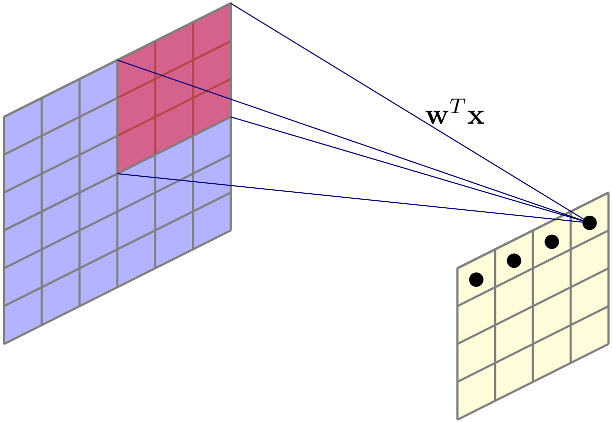

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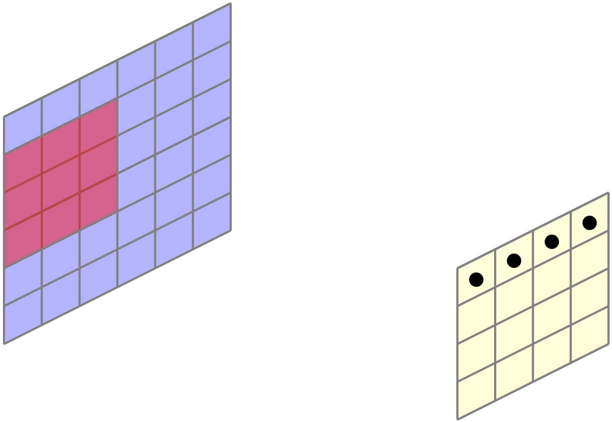

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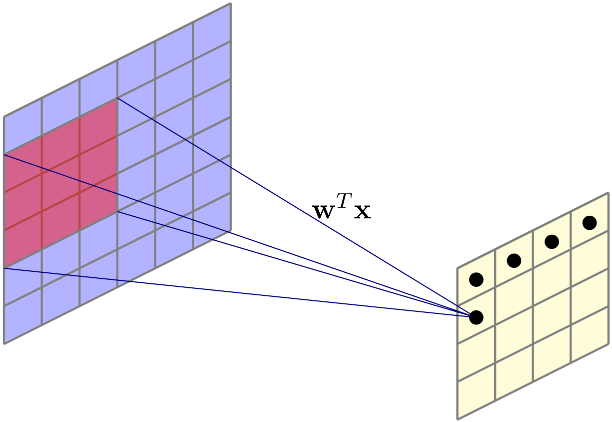

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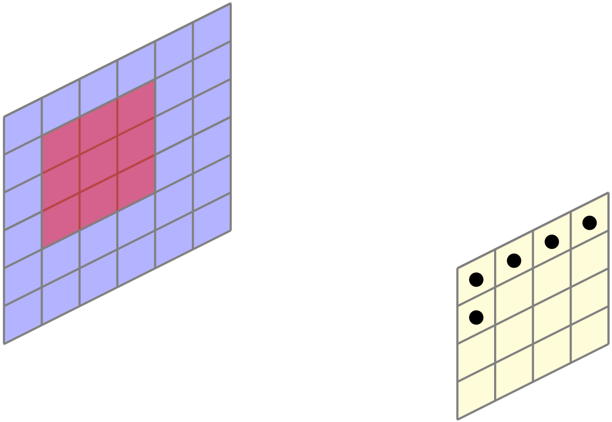

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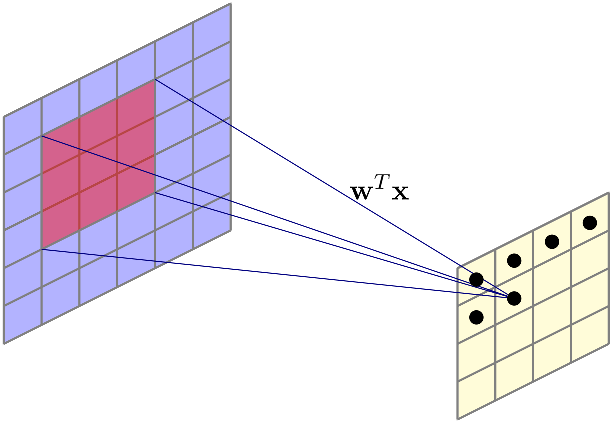

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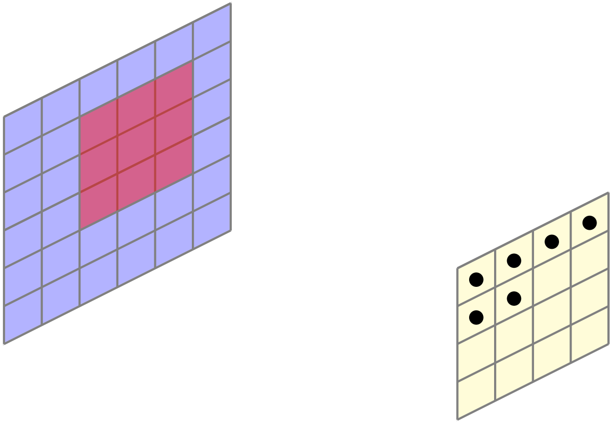

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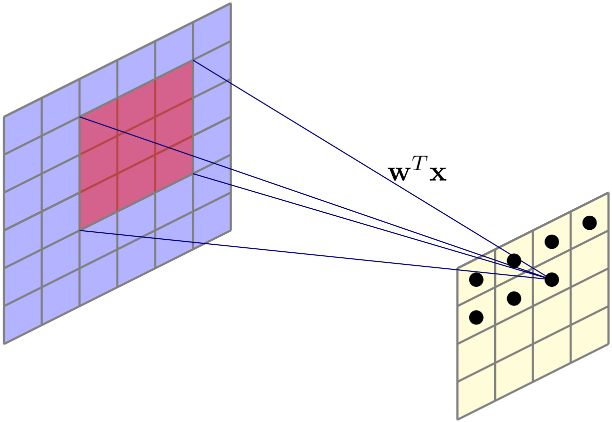

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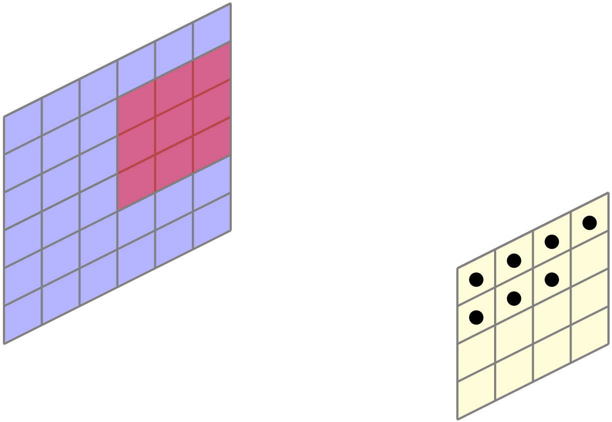

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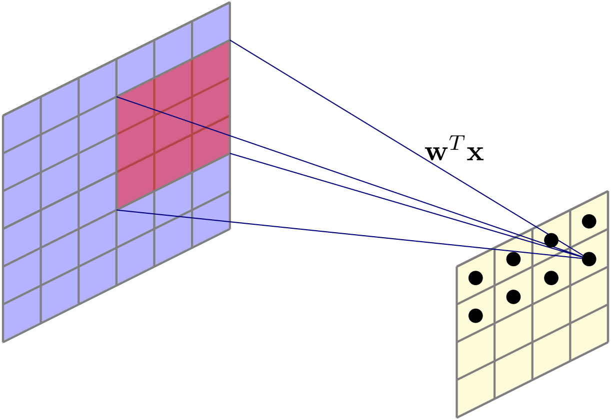

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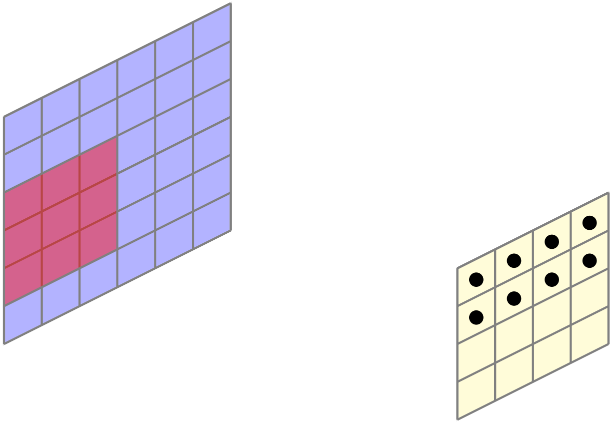

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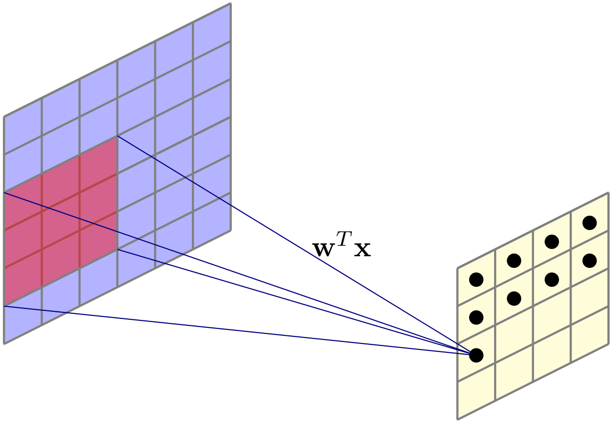

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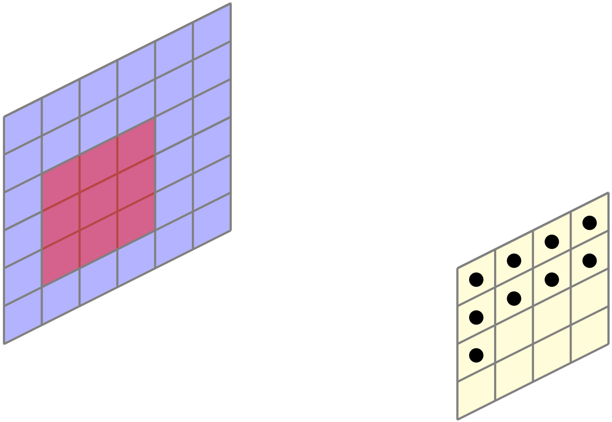

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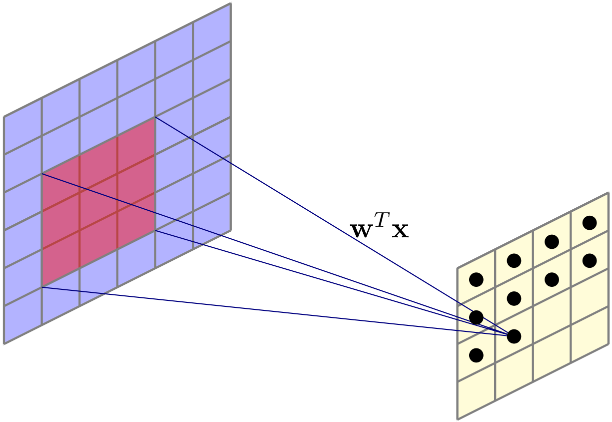

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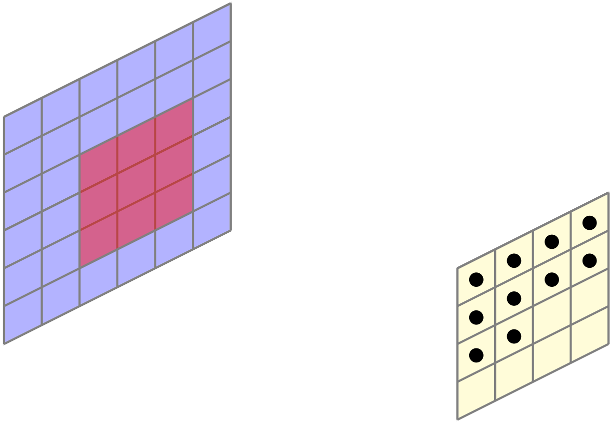

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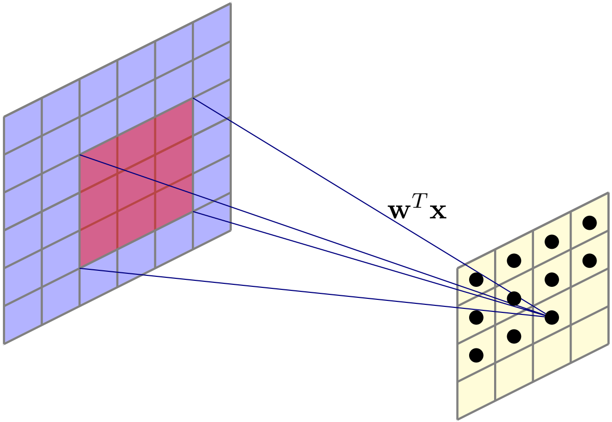

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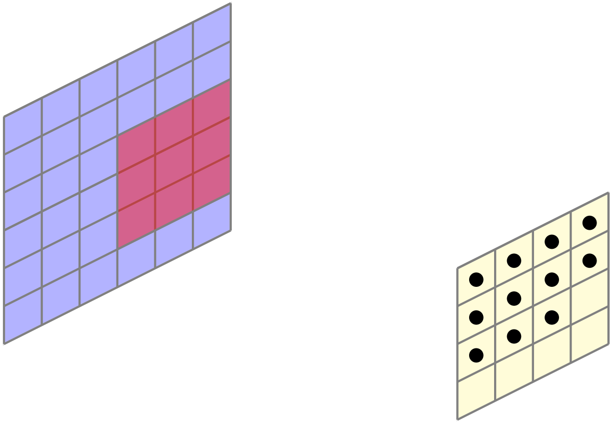

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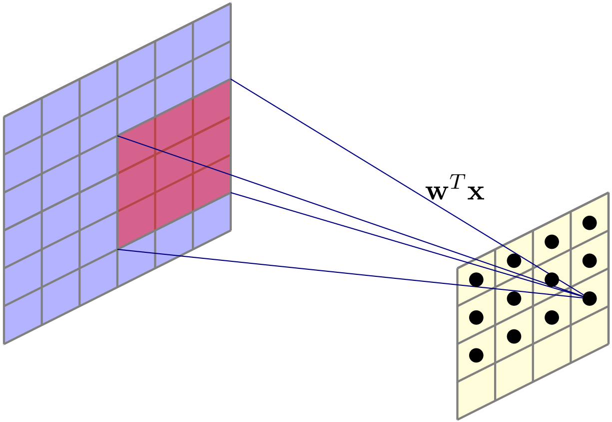

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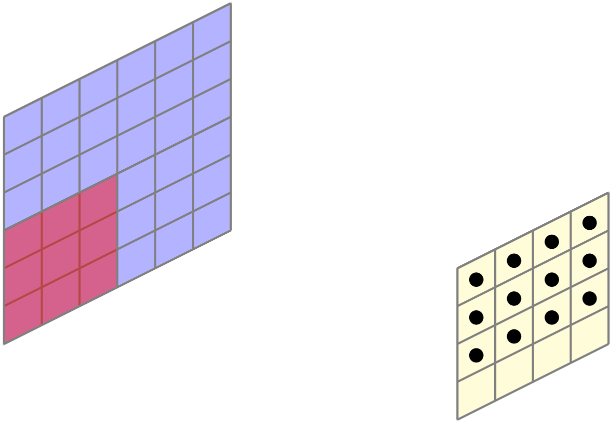

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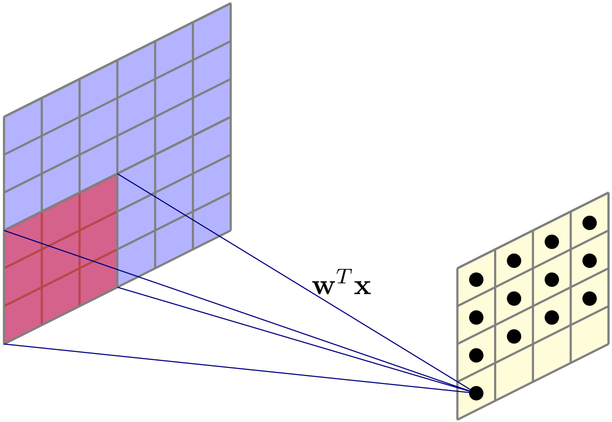

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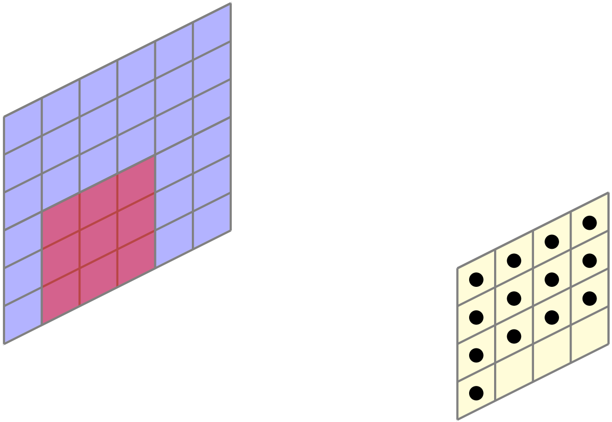

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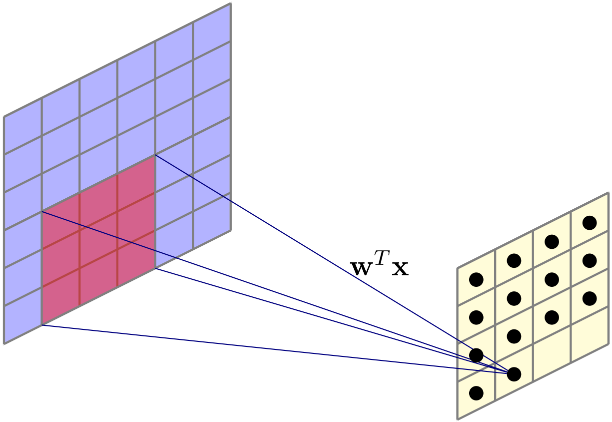

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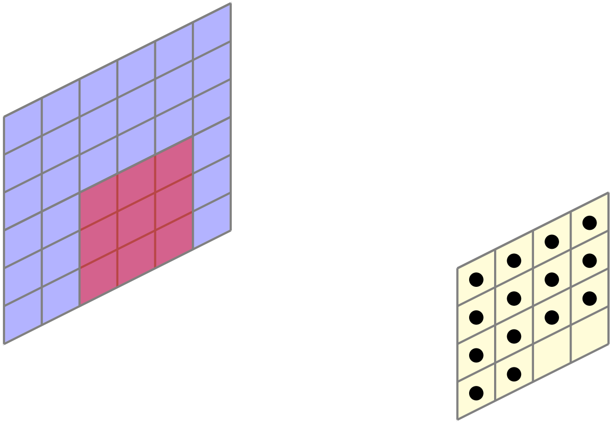

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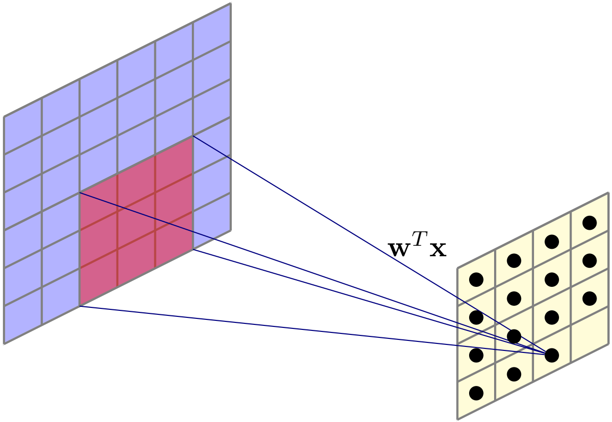

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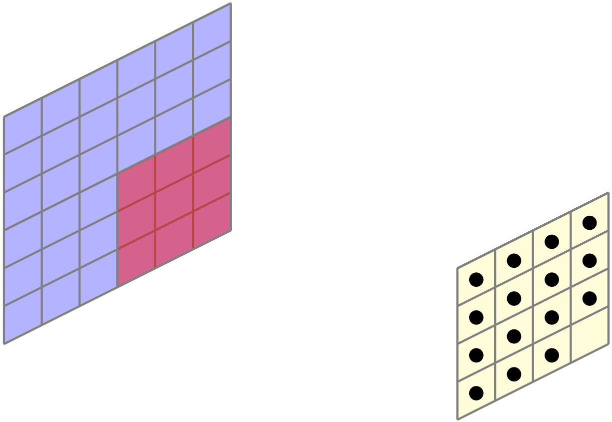

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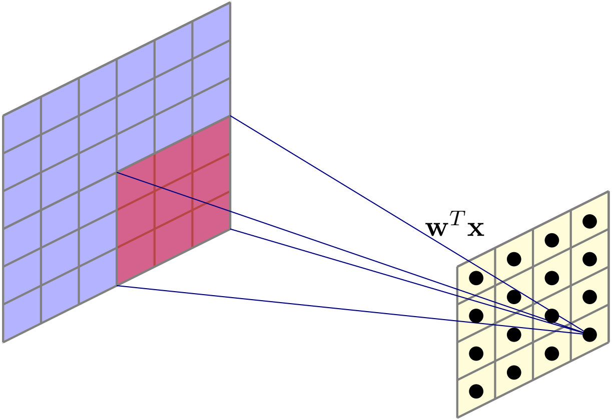

- What is the number of parameters?

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- In previous example: $N = 6, K = 3, S = 1$, Output size $= 4$
- For $N = 8, K = 3, S = 1$, output size is 6

Zero Padding

- Often, we want the output of a convolution to have the same size as the input. Solution: Zero padding.

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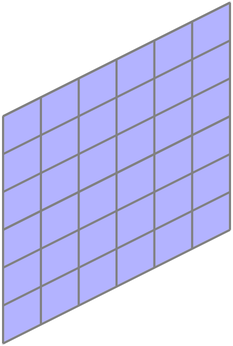
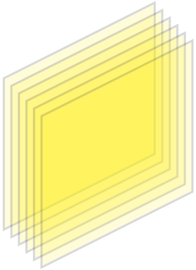
- In our previous example:

0	0	0	0	0	0	0	0
0							0
0							0
0							0
0							0
0							0
0							0
0	0	0	0	0	0	0	0

- Common to see convolution layers with stride of 1, filters of size K, and zero padding with $\frac{K-1}{2}$ to preserve size

Learn Multiple Filters





Learn Multiple Filters

- If we use 100 filters, we get 100 feature maps

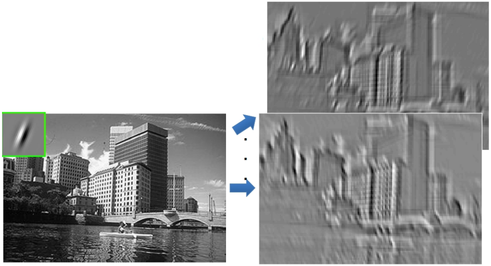


Figure: I. Kokkinos

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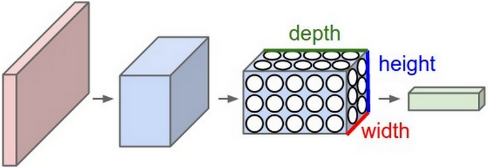

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  - Suppose input is of size $W_1 \times H_1 \times D_1$
  - Filter size is K and stride S
  - We obtain another volume of dimensions $W_2 \times H_2 \times D_2$
  - As before:

$$W_2 = \frac{W_1 - K}{S} + 1 \text{ and } H_2 = \frac{H_1 - K}{S} + 1$$

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- For convolutional layer:
  - Suppose input is of size $W_1 \times H_1 \times D_1$
  - Filter size is K and stride S
  - We obtain another volume of dimensions $W_2 \times H_2 \times D_2$
  - As before:

$$W_2 = \frac{W_1 - K}{S} + 1 \text{ and } H_2 = \frac{H_1 - K}{S} + 1$$

  - Depths will be equal

Convolutional Layer Parameters

Example volume: $28 \times 28 \times 3$ (RGB Image)

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For every filter: $3 \times 3 \times 3 + 1 = 28$ parameters

Convolutional Layer Parameters

Example volume: $28 \times 28 \times 3$ (RGB Image)
100 3×3 filters, stride 1
What is the zero padding needed to preserve size?
Number of parameters in this layer?
For every filter: $3 \times 3 \times 3 + 1 = 28$ parameters
Total parameters: $100 \times 28 = 2800$

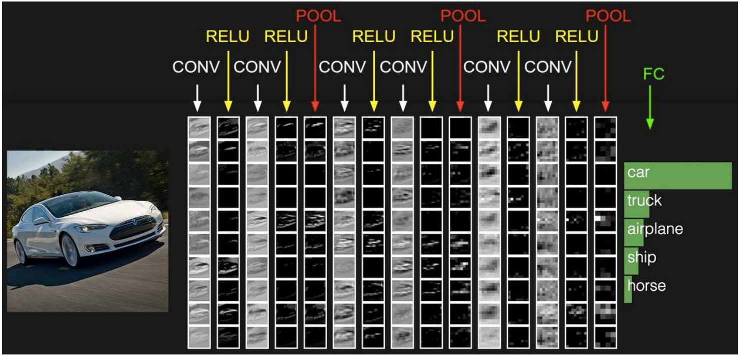
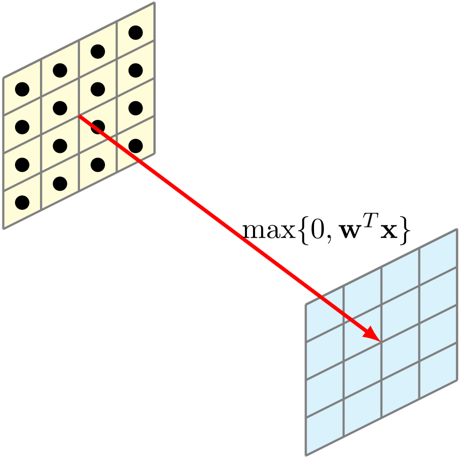


Figure: Andrej Karpathy

Non-Linearity



- After obtaining feature map, apply an elementwise non-linearity to obtain a transformed feature map (same size)

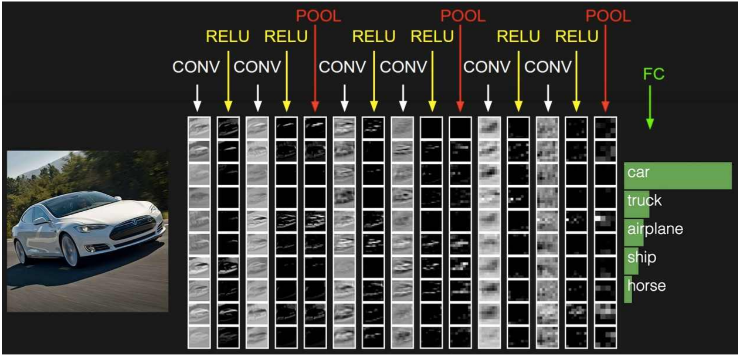
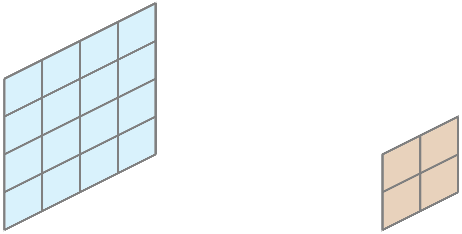


Figure: Andrej Karpathy

Pooling



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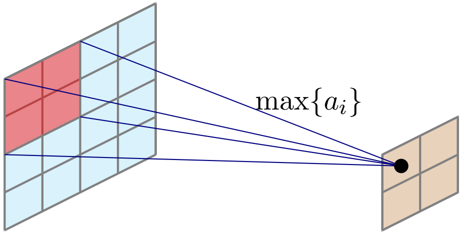

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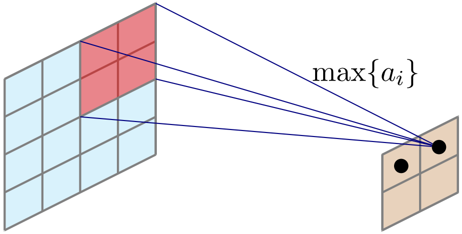

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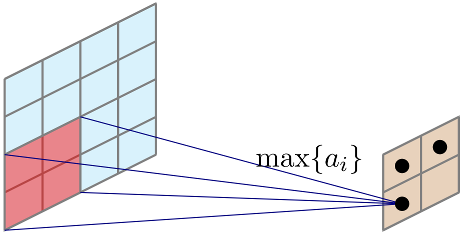

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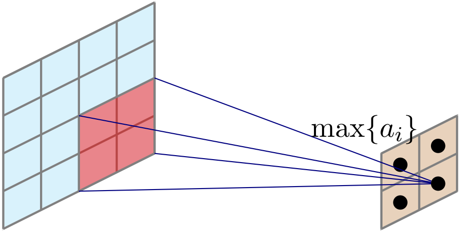
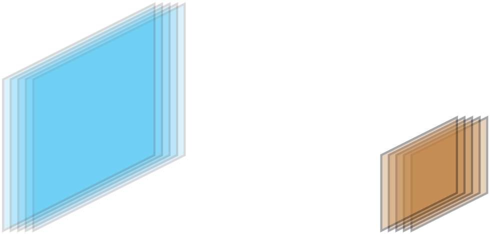


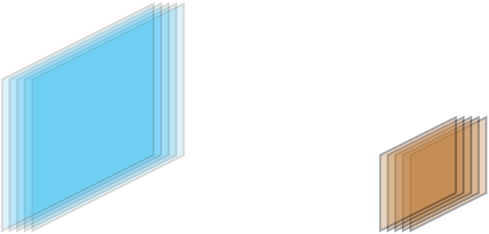
- Other options: Average pooling, L2-norm pooling, random pooling

Pooling



- We have multiple feature maps, and get an equal number of subsampled maps

Pooling



- We have multiple feature maps, and get an equal number of subsampled maps
- This changes if cross channel pooling is done

So what's left: Fully Connected Layers

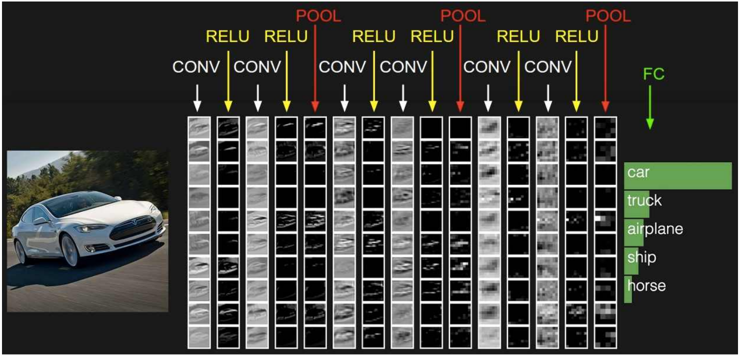
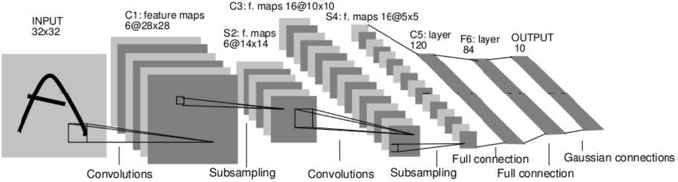


Figure: Andrej Karpathy

LeNet-5



- Filters are of size 5×5, stride 1

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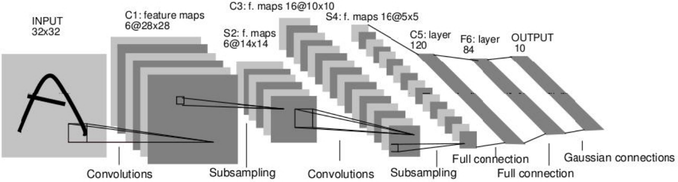

- Filters are of size 5×5, stride 1
- Pooling is 2×2, with stride 2

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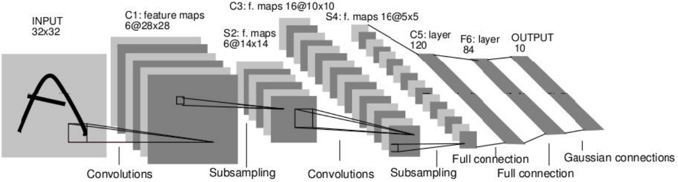
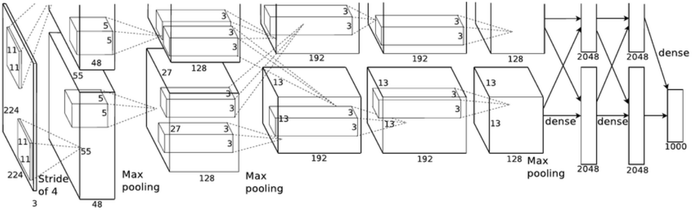


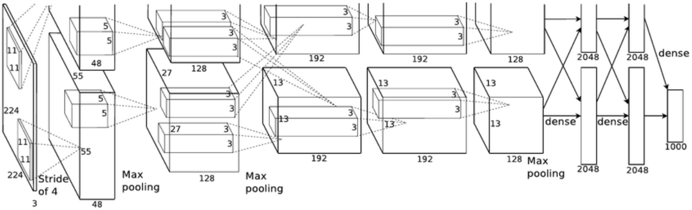
- Filters are of size 5×5, stride 1
- Pooling is 2×2, with stride 2
- How many parameters?

AlexNet



- Input image: 227 X 227 X 3
- First convolutional layer: 96 filters with K = 11 applied with stride = 4
- Width and height of output: $\frac{227-11}{4} + 1 = 55$

AlexNet



- Number of parameters in first layer?

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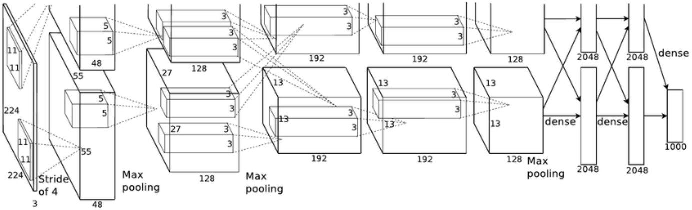
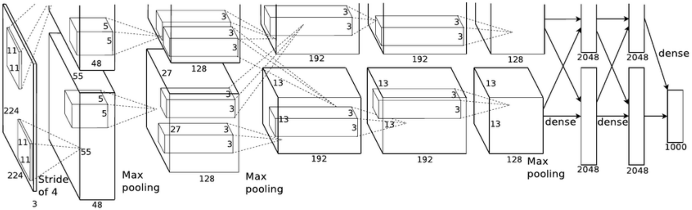


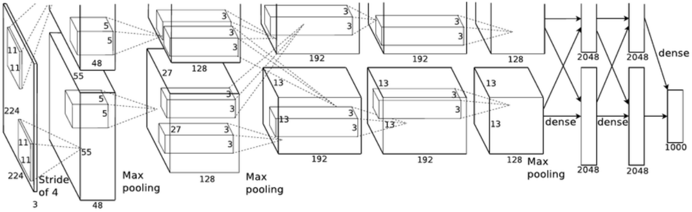
- Number of parameters in first layer?
- 11 X 11 X 3 X 96 = 34848

AlexNet



- Next layer: Pooling with 3 X 3 filters, stride of 2

AlexNet



- Next layer: Pooling with 3 X 3 filters, stride of 2
- Size of output volume: 27

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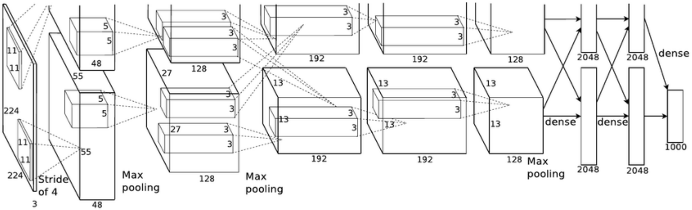

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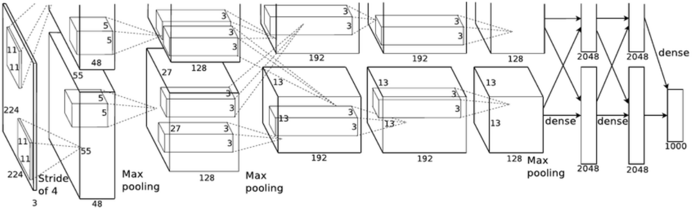
AlexNet



- Popularized the use of ReLUs

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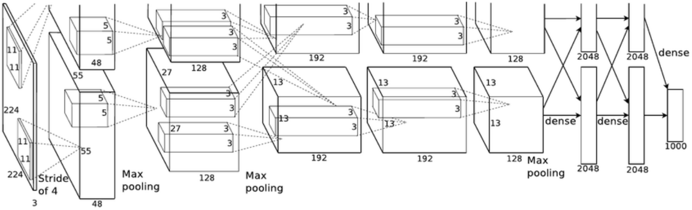

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- Used heavy data augmentation (flipped images, random crops of size 227 by 227)

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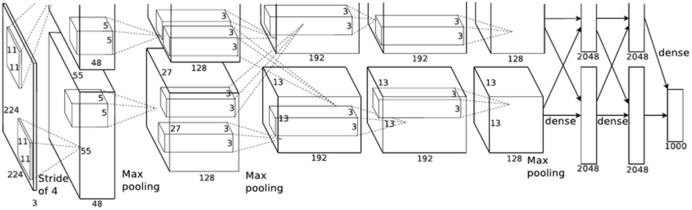
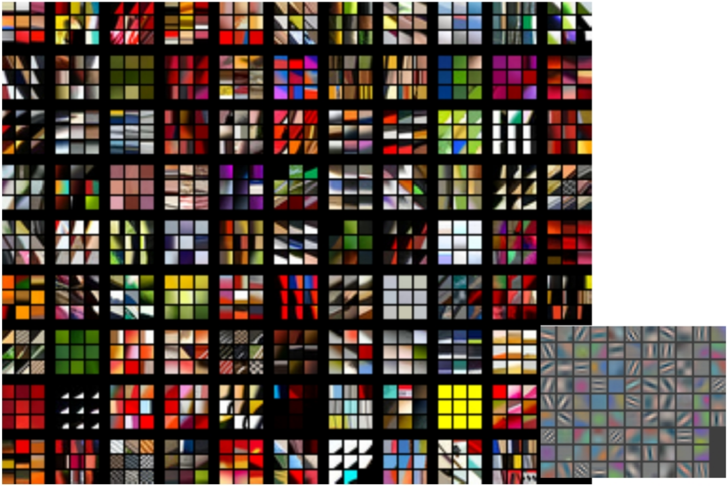


- Popularized the use of ReLUs
- Used heavy data augmentation (flipped images, random crops of size 227 by 227)
- Parameters: Dropout rate 0.5, Batch size = 128, Weight decay term: 0.0005 ,Momentum term $\alpha = 0.9$, learning rate $\eta = 0.01$, manually reduced by factor of ten on monitoring validation loss.

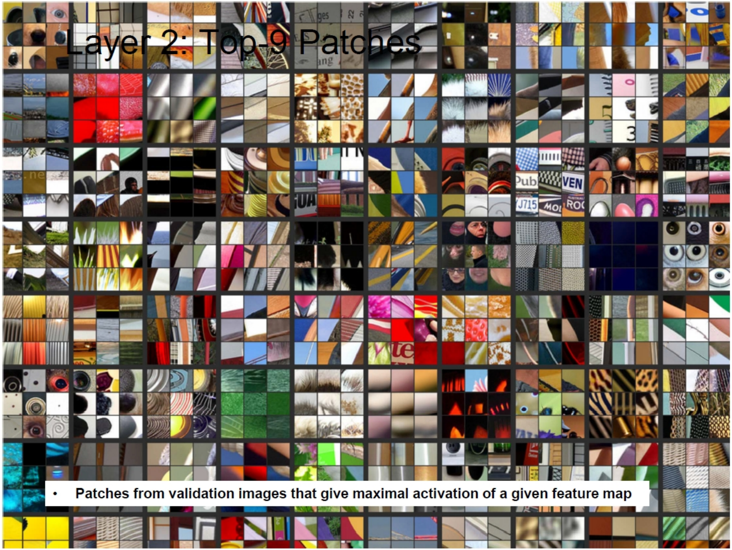
Short Digression: How do the features look like?

Layer 1 filters



This and the next few illustrations are from Rob Fergus

Layer 2 Patches



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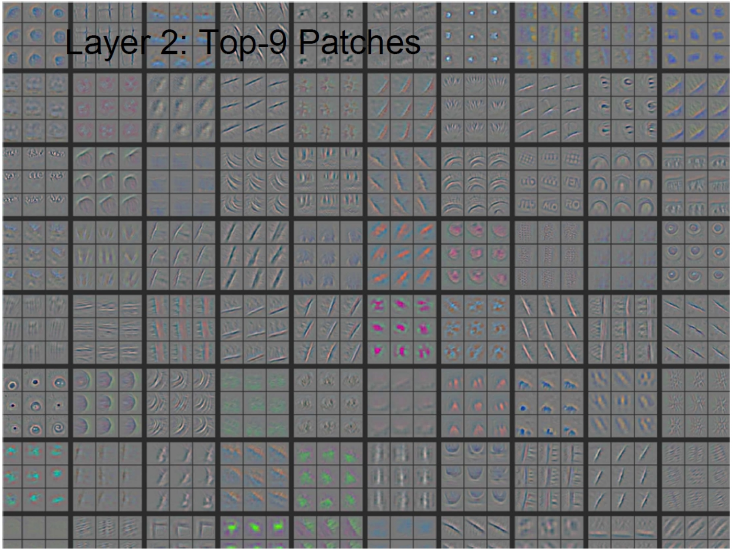
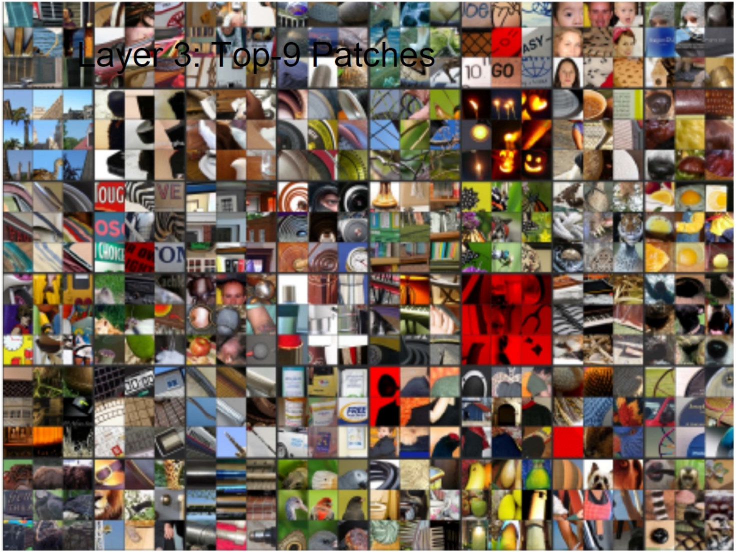


Layer 3 Patches



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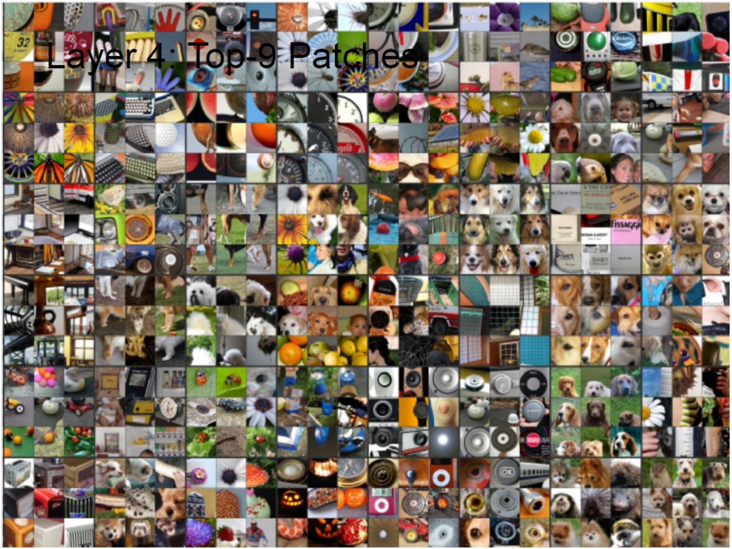
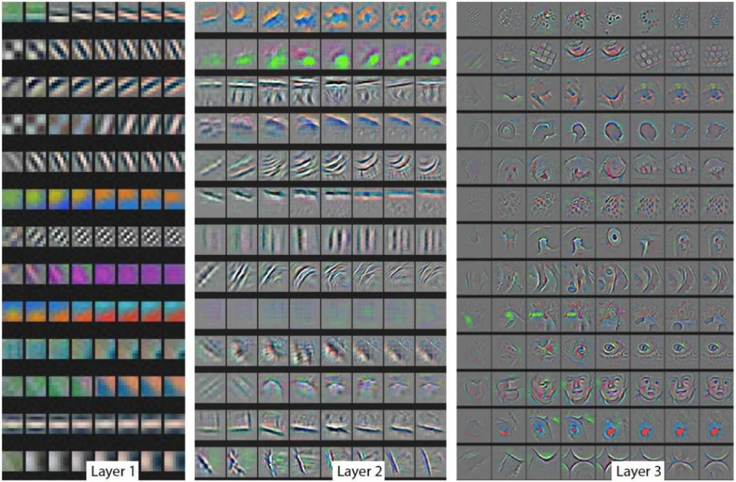
Layer 4 Patches



Layer 4 Patches

Evolution of Filters



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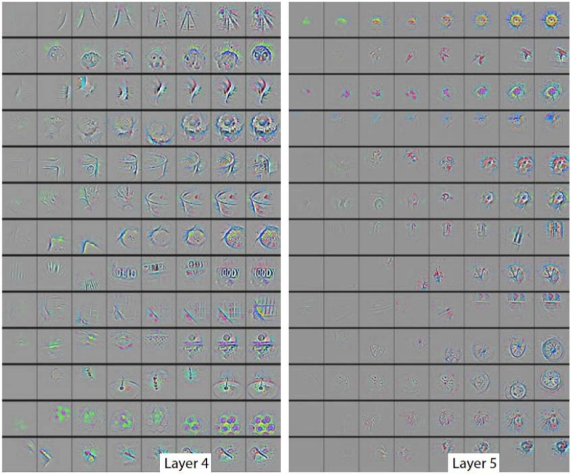

Caveat?

Back to Architectures

ImageNet 2013

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- ImageNet 2013: 14.8 % (reduced from 15.4 %) (top 5 errors)

VGGNet(Simonyan and Zisserman, 2014)

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

Table 2: **Number of parameters** (in millions).

Network	A,A-LRN	B	C	D	E
Number of parameters	133	133	134	138	144

- Best model: Column D.
- Error: 7.3 % (top five error)

VGGNet(Simonyan and Zisserman, 2014)

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  - Most parameters are in the fully connected layers

Going Deeper

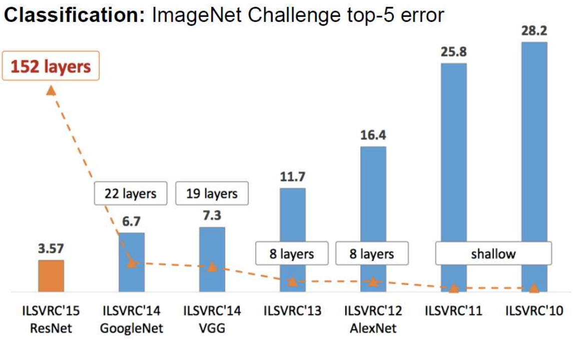
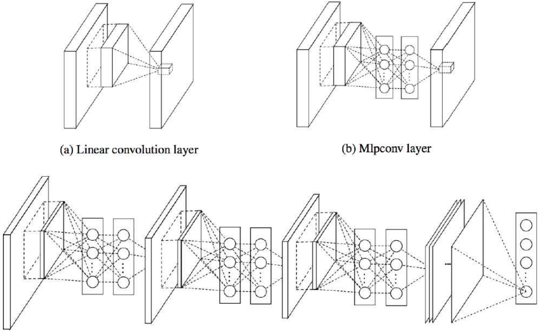


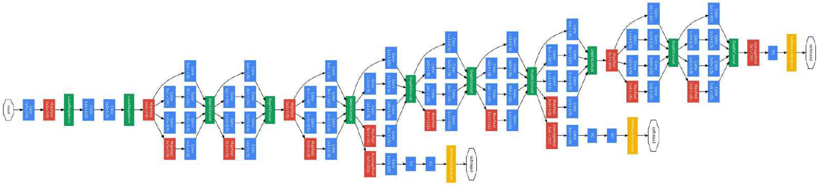
Figure: Kaiming He, MSR

Network in Network



M. Lin, Q. Chen, S. Yan, Network in Network, ICLR 2014

Google LeNet

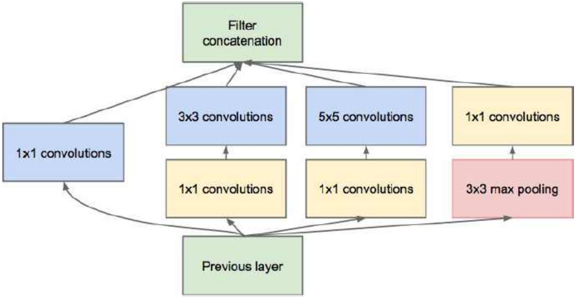


C.

Szegedy et al, Going Deeper With Convolutions, CVPR 2015

- Error: 6.7 % (top five error)

The Inception Module



- Parallel paths with different receptive field sizes - capture sparse patterns of correlation in stack of feature maps
- Also include auxiliary classifiers for ease of training
- Also note 1 by 1 convolutions

Google LeNet

type	patch size/ stride	output size	depth	#1×1	#3×3 reduce	#3×3	#5×5 reduce	#5×5	pool proj	params	ops
convolution	7×7/2	112×112×64	1							2.7K	34M
max pool	3×3/2	56×56×64	0								
convolution	3×3/1	56×56×192	2		64	192				112K	360M
max pool	3×3/2	28×28×192	0								
inception (3a)		28×28×256	2	64	96	128	16	32	32	159K	128M
inception (3b)		28×28×480	2	128	128	192	32	96	64	380K	304M
max pool	3×3/2	14×14×480	0								
inception (4a)		14×14×512	2	192	96	208	16	48	64	364K	73M
inception (4b)		14×14×512	2	160	112	224	24	64	64	437K	88M
inception (4c)		14×14×512	2	128	128	256	24	64	64	463K	100M
inception (4d)		14×14×528	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	3×3/2	7×7×832	0								
inception (5a)		7×7×832	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	1×1×1024	0								
dropout (40%)		1×1×1024	0								
linear		1×1×1000	1							1000K	1M
softmax		1×1×1000	0								

C. Szegedy et al, Going Deeper With Convolutions, CVPR 2015

Google LeNet

- Has 5 Million or 12X fewer parameters than AlexNet

Google LeNet

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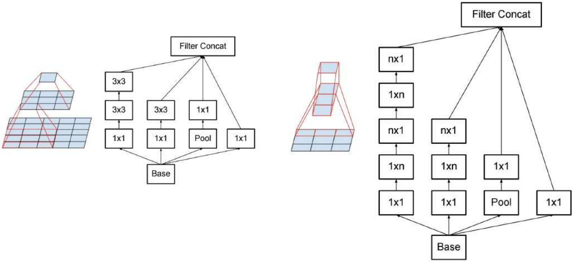
- Gets rid of fully connected layers

Inception v2, v3



C. Szegedy et al, Rethinking the Inception Architecture for Computer Vision, CVPR 2016

- Use Batch Normalization during training to reduce dependence on auxiliary classifiers
- More aggressive factorization of filters

Why do CNNs make sense? (Brain Stuff next time)

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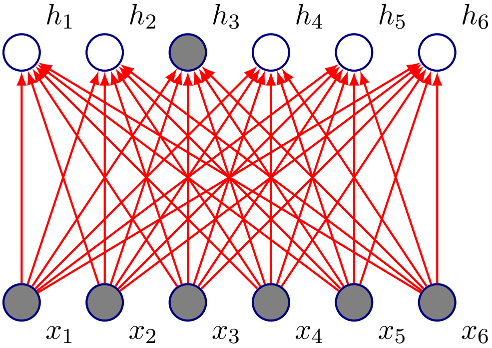
  - Convolutional networks have *sparse interactions* by making kernel smaller than input
  - $\implies$ need to store fewer parameters, computing output needs fewer operations ($O(m \times n)$ versus $O(k \times n)$)

Motivation: Sparse Connectivity



- Fully connected network: h_3 is computed by full matrix multiplication with no sparse connectivity

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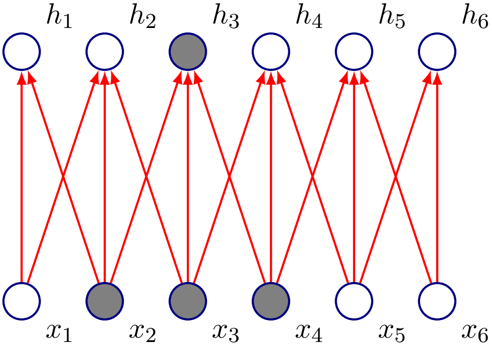

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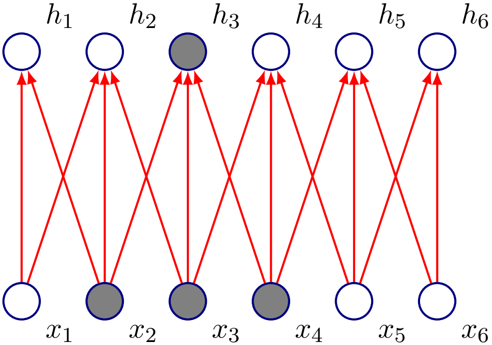

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- h_3 only depends on x_2, x_3, x_4

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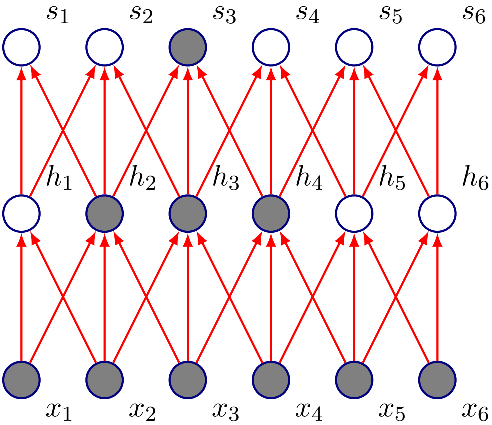

- Connections in CNNs are sparse, but units in deeper layers are connected to all of the input (larger receptive field sizes)

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- Storage improves dramatically as $k \ll m, n$

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- The form of parameter sharing used by CNNs causes each layer to be equivariant to translation
- That is, if g is any function that translates the input, the convolution function is equivariant to g

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- Convolution is not equivariant to other operations such as change in scale or rotation

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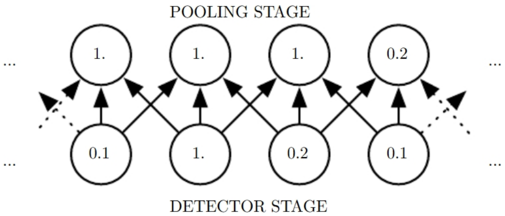
- If input is translated by small amount: values of most pooled outputs don't change

Pooling: Invariance



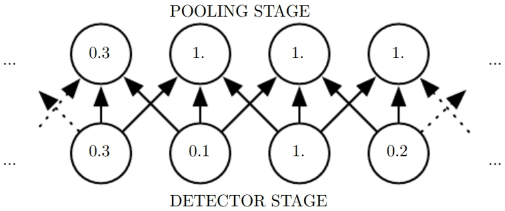


Figure: Goodfellow *et al.*

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- Features can learn which transformations to become invariant to (Example: Maxout Networks, Goodfellow *et al* 2013)
- One more advantage: Since pooling is used for downsampling, it can be used to handle inputs of varying sizes

Next time

- More Architectures
- Variants on the CNN idea
- More motivation
- Group Equivariance
- Equivariance to Rotation

Quiz!